Implicit differentiation

A formula for Implicit differentiation

If $F_y \neq 0$ & $F(x,y)=0$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Ex) $y^2 - x^2 - \sin(xy) = 0$

Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{-2x - y\cos(xy)}{2y - x\cos(xy)}$$

$$= \frac{2x + y\cos xy}{2y - x\cos xy}$$

$\therefore$ We can say

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} \quad \& \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Ex) Find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ at $(0,0,0)$

if $x^3 + z^2 + y e^{xz} + z\cos y = 0$

Let $F(x,y,z) = x^3 + z^2 + y e^{xz} + z\cos y$

$F_x = 3x^2 + zye^{xz}$

$F_y = e^{xz} - z\sin y$

$F_z = 2z + xye^{xz} + \cos y$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{3x^2 + zye^{xz}}{2z + xye^{xz} + \cos y}\Bigg|_{(0,0,0)} = 0$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(e^{xz} - z\sin y)}{2z + xye^{xz} + \cos y} = -1$$

Ex) $w = \ln(x^2+y^2+z^2)$, $x = \cos t$
$y = \sin t$
$z = 4\sqrt{t}$

Find $\frac{dw}{dt}$ at $=3$.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}\Bigg|_{t=3}$$

$$= \left(\frac{2x}{\quad}\right)\cdot(-\sin t) + \left(\frac{2y}{\quad}\right)(\cos t) + \left(\frac{2z}{\quad}\right)\cdot \frac{2}{\quad}$$

$$(x^2+y^2+z^2) \qquad + (x^2+y^2+z^2) \qquad (x^2+y^2+z^2)\sqrt{t}$$

$$= \frac{-2x(\sin t) + 2y\cos t + 4z\left(t^{-\frac{1}{2}}\right)}{x^2+y^2+z^2}$$

$$= \frac{-2(\cos t)\sin t + 2\sin t\cos t + 16\sqrt{t}\cdot t^{-\frac{1}{2}}}{x^2+y^2+z^2}$$

$$= \frac{16}{\underline{\sin^2 t+\cos^2 t} + 16t} = \frac{16}{1+16t}\Bigg|_3$$

$$= \frac{16}{49}$$

Easy way $\qquad x, y, z$

$$w = \ln(x^2+y^2+z^2)$$

$$w = \ln(\sin^2 t + \cos^2 t + 16t)$$

$$w = \ln(1+16t)$$

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$$w' = \frac{16}{1+16t}\Bigg|_{t=3} = \frac{16}{49}$$

Ex) $w = z - \sin xy$, $x = t$
$y = \ln t$
$z = e^{t-1}$

$$\frac{\partial w}{\partial t}\Bigg|_{t=1} = \,?$$

$$w = e^{t-1} - \sin(t \ln t)$$

$$w' = e^{t-1} - \cos(t\ln t)\left[\ln t + 1\right]\Bigg|_{t=1}$$

$$= 1 - 1[1] = 0$$

Ex) Find $\frac{\partial w}{\partial r}$ when $r=1$, $s=-1$

if $w = (x+y+z)^2$, $x = r-s$
$y = \cos(r+s)$
$z = \sin(r+s)$

$$W = \left((r-s) + \cos(r+s) + \sin(r+s)\right)^2$$

$$W' = 2\left((r-s) + \cos(r+s) + \sin(r+s)\right) \cdot \left[1 - \sin(r+s) + \cos(r+s)\right]$$

$$W'\Big|_{r=1, s=-1} \quad 2\left[(1+1) + \cos 0 + \sin 0\right]\left[1 - \sin 0 + \cos 0\right]$$

$$2[3][2] = 12$$

Find $\frac{\partial z}{\partial u}$ & $\frac{\partial z}{\partial v}$ when $u=1$, $v=-2$

if $z = \ln q$ & $q = \sqrt{v+3}\ \tan^{-1} u$

$$(\tan^{-1} u)' = \frac{1}{1+u^2}$$

$$\frac{\partial z}{\partial u} = \frac{dz}{dq} \cdot \frac{dq}{du}$$

$$\frac{1}{q} \cdot \sqrt{v+3} \cdot \frac{1}{1+u^2}$$

$$\frac{\sqrt{v+3}}{q \cdot (1+u^2)}$$

$$\frac{\partial z}{\partial v} = \frac{dz}{dq} \cdot \frac{\partial q}{\partial v}$$

$$\frac{1}{q}\left(\frac{\tan^{-1} u}{2\sqrt{v+3}}\right)$$

$$\frac{\cancel{\tan^{-1}} u}{\sqrt{v+3}\ \cancel{\tan^{-1}} u\,(2\sqrt{v+3})}$$

$$1 \qquad 1$$

$$\frac{\cancel{\sqrt{v+3}}}{\cancel{\sqrt{v+3}} \cdot \tan^{-1} u \cdot (1+u^2)}$$

$$\left.\frac{1}{\tan^{-1} u \cdot (1+u^2)}\right|_{u=1,\, v=2}$$

$$\frac{1}{\tan^{-1}(1) \cdot (2)} = \frac{1}{\frac{\pi}{4} \cdot 2} =$$

$$\frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$\left.\frac{}{2\,(v+3)}\right|_{\substack{u=1 \\ v=-2}}$$

$$\frac{1}{2\,(-2+3)} = \frac{1}{2}$$

Find $\frac{dw}{dt}$ interms of t when $t=\frac{1}{2}$

$w = \ln(x^2+y^2+z^2)$

$x = \cos t$

$y = \sin t$

$z = 4\sqrt{t}$

$$w = \ln(\cos^2 t + \sin^2 t + 16t)$$

$$= \ln(1 + 16t)$$

$$w' = \left.\frac{16}{1+16t}\right|_{t=\frac{1}{2}} = \frac{16}{1+8} = \frac{16}{9}$$